A Computational Simulation of bending fatigue in spur gear with profile modification

Une simulation numérique de la fatigue en flexion d'engrenage à denture droite avec modification de profil

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ملخص:

في هذا العمل اجرينا نمذّجة باستخدام طريقة العناصر المحدودة (ANSYS MEF) لتحديد عمر المرحلة الولى للكلل (Ni) فالتروس المسننة المستقيمة و المصنوعة من الصلب (42 CD 4) , ثم استخدمنا برنامج اضافي للعناصر المحدودة (FRANC 2D) الذي يعتمد على مبدأ ميكانيك القطع الخطية الرجعية لمحاكاة انتشار الشقوق عند جذر السن هذا الاخير يعتمد على معادلة (Paris-Erdogan) يقدم النموذج المستخدم طريقة لتحديد بداية التشققات بالإضافة الى سرعة انتشار هذهالأخيرة. النتائج المتحصل عليها تم مقارنتها بمختلف الاجهادات المطبة لمراقبة تأثير درجة الاجهادات على المدة الكاملة للكلل. كما ان هذه الدراسة مفيدة للغاية و تسمح بانجازو تطوير خطط التفتيش الخاصة بالتروس المدروسة.

Abstract :

In this work, we conducted a modeling using the finite element method (FEM ANSYS) for determining the lifetime of Ni initiation of a crack in the straight toothed gears in 42CrMo4 steel. An additional program of finite element FRANC2D based on the principle of LEMF was used to simulate the propagation of cracks at the tooth root. The latter uses the Paris-Erdogan equation for the growth of cracks. The model presented is used to determine the propagation velocity of the crack. Acquired results we they allowed to understand the effect of the charge load on the life of spur gears on one hand, of besides part also to have the state of stresses on the crack-tip and the speed of crack propagation. This study gives the possibility of working out a plan of examination for these gears.

Key words: Simulation; Gear; Fatigue; Crack; Bending

Résumé :

Dans ce travail nous avons effectué une modélisation par la méthode des éléments finis pour la détermination de la durée de vie de l'initiation Ni d'une fissure dans les engrenages droits en acier 42CrMo4. Un programme supplémentaire des éléments finis FRANC2D basé sur le principe de la MELR a été utilisé pour simuler la propagation de fissures à la racine de dent. Ce dernier utilise l'équation de Paris-Erdogan. Le modèle présenté est exploité pour déterminer la vitesse de propagation de la fissure. Les résultats obtenus nous on permit de comprendre l'effet de l'état de chargement sur la durée de vie d'un engrenage d'une part, d'outre part aussi d'avoir l'état de contrainte au voisinage de fissure et la vitesse de propagation de fissure. Cette étude offre la possibilité d'élaborer un plan d'inspection pour ces engrenages.

Mots Clés: Simulation; Engrenage ; Fatigue; Fissure ; Flexion

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1. INTRODUCTION

Gears are the main components of systems and mechanisms and are subjected to failure by fatigue. This is especially important in the industrial context of today, where the continuous search for optimization requires the use of the machine elements where the loading levels are near the critical limits. It is therefore essential to understand their behavior under various loading levels to avoid unexpected breakdowns with disastrous consequences. Since the gear teeth, in turn, undergoes the load of transmitted power, their state of stress varies cyclically, thus explaining their fatigue and failure at loading level is well below the yield strength of the material. Specifically, the breakdown of the gears frequently results from a spread of tooth root cracks caused by bending fatigue because it is responsible for more than 30%. Consequently, gear systems failures generate substantial losses. It becomes essential to understand this phenomenon in order to minimize negative impacts. Several standardized conventional procedures, for example, the DIN, AGMA, ISO, etc. [1-3], may be used to approximate the determination of the capacity of resistance of a gear tooth root. They are generally based on the comparison between the maximum stresses at the tooth root with the permissible bending stress. Their determination depends on some factors that allow a proper consideration of actual working conditions (additional internal dynamics and external forces, contact area, material, surface roughness, etc.). Conventional methods are exclusively based on experimental tests of the reference equipment (FZG). According to this [2], only the gear hardness with maximum stress at the root of the tooth changes the life of fatigue, and they consider that the final phase of the fatigue process in the root of the tooth gear, i.e the appearance of the final rupture. In [4], the authors study the equations of rack cutters for generating involutes gears with asymmetric teeth. In [5],the authors used the undercutting equation, for determining a minimum number of teeth of the spherical pinion.

However, the whole process of failure by fatigue of mechanical elements can be divided into the following steps (a) Micro-crack 'Nucleation'; (b) Growth in the short-crack; (c) Growth in the long crack and (d) occurrence of the final failure [6]. In engineering applications, the first two steps are generally referred to as the crack-initiation period, while the longcrack growth is called crack growth period.

But, in modern gear practice and manufacturing, the majority of gear applications are covered by the standard 20° in volute teeth generated by rack, hob, and CNC cutting process. This has many advantages such as: interchangeability, insensitivity to change in nominal center distance, commercial availability, and easy manufacturing by conventional methods (i.e., hobbing) [7].

In several cases, the correction of gears is made for technical reasons such as the correction of the center distance in order to avoid interference at the level of contact through clenched teeth. For it, the modification of gears allows to make easier the gear mesh and sometimes to increase the mesh stiffness [8]in the case of a positive modification. On the contrary, when they have a negative modification, the mesh stiffness reduces [9]. In undercutting, the tooth filet is generated as the tip of the cutter removes material from the in volute profile, thus resulting in teeth that have less tooth thickness at the root, where the critical section is usually located.

This work is specifically aimed at the problem in bending fatigue of the gears (with modification) and the propagation of the crack.Our aim is to establish an approach for the design of the gear on the Linear Elastic Fracture Mechanics (LEFM) base using the finite element method, due to the complexity of the shape of the gear tooth, to predict the initiation and propagation paths of the crack.

2 FEM BENDING FATIGUE MODELING

A number of internal sources may cause the excitation of a gear directly in the meshing area such as the impact of the mesh initiation and the tooth stiffness. Therefore, the variation in some parameters is evident namely the geometrical deviations of teeth bearings strain and shafts. Significant changes in tooth stiffness could be the consequence of fatigue crack in the tooth root.

Throughout the cracks progression, the tooth bending rigidity decreases. This tends to overload the adjacent teeth and promotes the crack initiation. Systematically, with bending fatigue, regular breaking in several successive teeth occurs. Premature failure of the gear teeth can be caused, basically, by a poor design, incorrect mounting, material defects or overloads. The entire process of bending fatigue by failure

is the sum of the crack initiation and the propagation up to break (Eq. 1).

$$N_f = N_i + N_p \quad \text{(Eq. 1)}$$

Commonly N_i is higher than N_p but surface treatment affects the behavior in fatigue. For shot-penned gears, N_p can represent 15 to 30% of N_f and 30 to 40% for carburized[10].Also, the induction hardening is used to harden specific areas of a work piece without affecting thematerial properties of the part as a whole[11]. Moreover, a gear subjected to a small load will have the maximum of its life in service in the initiation phase. However, the propagation up to break phase will be more significant during high loading levels (Fig. 1).

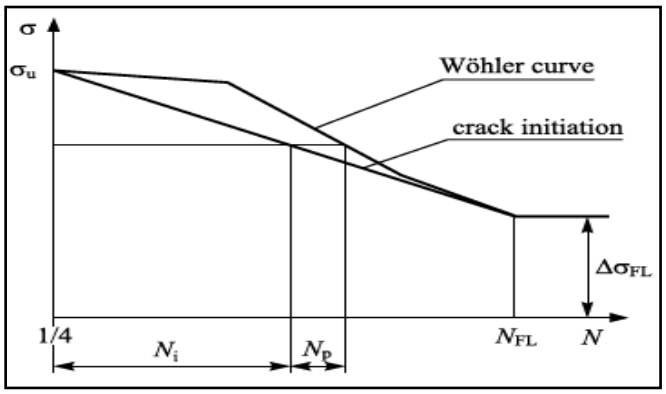


Figure 1 Schematic representation of the service life of mechanical elements [13]

2.1. First step: Fatigue crack initiation model

The number of cycles required to initiate a crack at a notch root of the tooth may be estimated by several methods. The first step is to determine the stress and strain at the critical region. The second step is to account for the damage accumulation at the critical location (the root of the tooth) by a means damage criterion.

The initiation of the bending cracks is located in the fillet of the teeth on the active side(fig.3.a), where the cyclic stresses in tensile are maximal. From literature[3], the progression of these cracks takes place towards a point corresponding to zero stress which is located initially near the root radius in the center of the tooth. This point moves until it reaches the fillet on the other side of the tooth. The generated trajectory refers to the direction with the least resistance to propagation.

Recent investigation by[12], attempts were made to determine damage at a notch root indirectly by using the nominal applied stress or strain and fatigue concentration factor. More recent approaches were developed by[13]to use the local stress and strain to determine the damage accumulation.

The present fatigue crack initiation model is based on the of continuum mechanics approach. The material is homogeneous and isotropic, i.e. assumed without imperfections or damage. A brief review highlighted the method used to predict the crack initiation life under constant amplitude cyclic loading, based on the relationship of Coffin-Manson(Eq. 2) [14] strains (ε), stresses (σ) and loading cycles number (N_i).

However, the method of deformation versus a number of cycles($\varepsilon - N_i$) is usually used to determine the number of cycles required for the initiation of fatigue cracks, where it is assumed that the crack is initiated at the point of greatest stress.

The total strain gradient $\Delta\varepsilon$ has two components, elastic ($\Delta\varepsilon_e$) and plastic ($\Delta\varepsilon_p$) strain gradients, described by the equation below:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\Delta\sigma}{2E} + \varepsilon_f^{'}\left(\frac{\Delta\sigma}{2\sigma_f^{'}}\right)^{\frac{1}{n'}} = \frac{\sigma_f^{'}}{E}(2N_i)^b + \varepsilon_f^{'}(2N_i)^c \qquad \text{(Eq. 2)}$$

Initiation of fatigue cracks characterizes one of the most important steps in the process of fatigue. The position and the mode of fatigue cracks initiation depend on the microstructure of the studied material, the type of applied stress and the geometry of the test piece. The phase of initiation of fatigue in a virgin material is often supposed to be the growth of short cracks until the size a_{th}(Fig. 2), which is the transition length from short to long cracks.

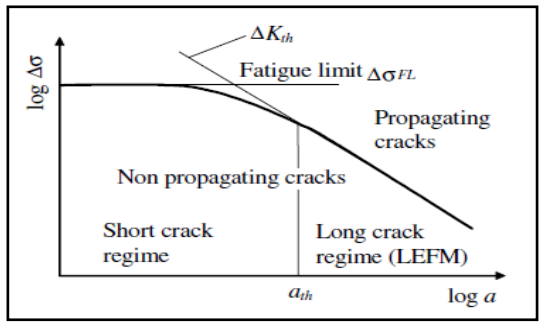
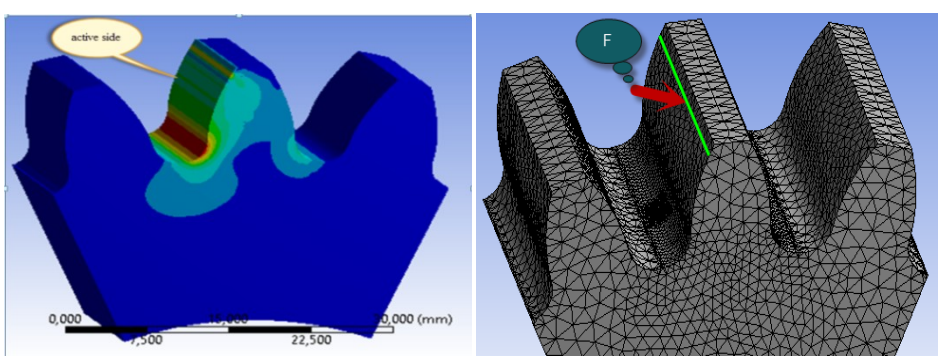


Figure 2: Schematic of Kitagawa–Takahashi of crack growth evolution and transition from short to long[12]

Crack propagation analysis begins with the proper configuration of sizing, orientation, and position. Depending on the wheels geometry, it has been shown that the cited parameters were important in the crack propagations [12]. A finite element model for the initiation of crack under bending fatigue in the tooth root is presented in the present investigation. The model, for N-deformation, calculates driver at the tooth root for the crack initiation phase shown in (Fig. 1). The gear tooth loaded is subjected to the normal force *F*/*b* (N/mm), as displayed in (Fig. 3.b &Fig. 4), applied to the highest point of the single tooth contact (HPSTC) [13].The normal load (F/b) variation with angle rotation is shown in (Fig.4).Basic data, geometric, properties and mechanical of studied gears are presented in table 1.



a) b)

Figure.3: FEM of 3 teeth, a) geometric and stress distribution, b) mesh, refinement and loading of the tooth.

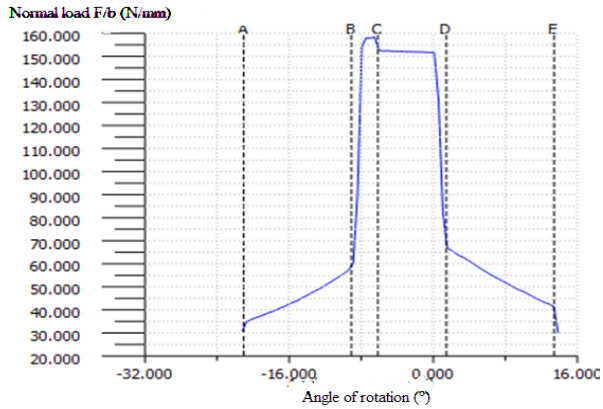


Figure.4: The normal load (linear load) 2D curve

Tableau.1: Properties and geometric features mechanical (Basic data of studied gears)

	Gear1	Gear2	Gear1	Gear2
Teeth number z	16	24	Poisson ratio	$\nu = 0.3$
Module m (mm)	4.5	4.5	Coefficient of fatigue force	σ'_f=1820 MPa
Center distance a (mm)	90		Axial fatigue strength coefficient	ε'_f= 0.65
Pressure angle °	20°		Exponent force	b = 0.08
Offset Coefficient	+0.190	-0.190	Fatigue ductility exponent	c = - 0.76
Tooth width b (mm)	44	40	Cyclic strain hardening exponent	n' = 0.14
Material	42CrMo4		Paris constant (m)	m=4.16
Elasticity modulus	E = 2.06 x10^5MPa		Paris constant (c)	C=3,31.10^{17}

2.2. Second step: Fatigue crack growth model

In the previous section, we discussed the requirements for crack initiation from notches. The growth of this initiated crack will be the subject of this section.

The application of Linear Elastic Fracture Mechanics (LEFM) fatigue is based on the assumption that the growth rate of fatigue cracks, *da/dN*, is a function of the stress intensity gradient $\Delta K = (K_{max} - K_{min})$ where (*a*) is the length of the crack and *N*is the number of loading cycles. In this study, the equation of *Paris-Erdogan* [15](Eq.3) is used to describe the crack growth rate, where C and m (Tab.1) are material parameters. Regarding the propagation delay of the crack, one can obtain the number N_p of loading cycles of rupture of the tooth from the integration of equation (Eq. 4) indicating that the required number of loading cycles for a crack propagating from the initial length a_{th} and a_c critical crack length can be explicitly determined, if C, m, and ΔK are known.

C and m are important parameters and can be obtained experimentally, usually by means of a three-point bending test according to ASTM E 399-80.

The knowledge of different cracks regime, predicting their propagation speed, cracking masterly direction and also important parameters like stress intensity factor (SIF) is essential for an optimized design of mechanical parts. SIF is an essential parameter of linear fracture mechanics in case of cracked structures with singular stress field to calculate the fatigue life [16].

For more complex geometry and loading, it is necessary to use alternative methods. In this work, the software finite element developed by the Cornell Fracture Group FRANC2D [12, 17] was used to simulate the propagation of fatigue cracks. A unique feature is the automatic FRANC2D ability to crack propagation.

$$\frac{da}{dN} = C[\Delta K(a)]^m \qquad \text{(Eq. 3)}$$

$$\int_0^{N_p} dN = \frac{1}{C}.\int_{a_{th}}^{a_c} \frac{da}{[\Delta K(a)]^m}$$

$$K_I = \frac{G}{K+1}.\sqrt{2.\frac{\pi}{L}}.[4(\upsilon_b - \upsilon_d) + \upsilon_e - \upsilon_c]$$

$$K_{II} = \frac{G}{K+1}.\sqrt{2.\frac{\pi}{L}}.[4(u_b - u_d) + u_e - u_c] \qquad \text{(Eq. 4)}$$

$$K = \frac{E}{2.(1+\nu)}$$

A geometrical model is created with the object solid program (OSM) (Fig.5.a).

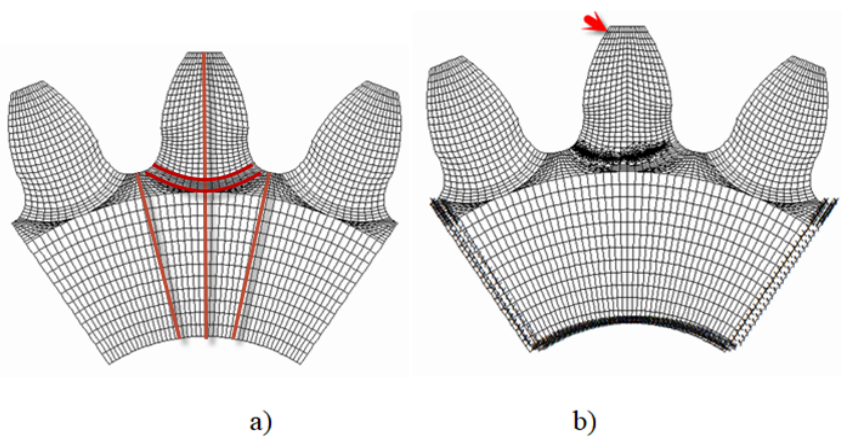


Figure.5 (a) Model of gear,(b) Boundary element.

The crack growth analyses were performed where the maximum stresses occur in a gear tooth root. Although the opening of the crack due to peak stress concentrations may appear on both sides of the tooth root, the most critical for the spread is the active side of the tooth root (Fig.5.b). The maximum principal stress distribution (tensile stress) in the discharge of the tooth is given in (Fig. 6). The location in the root zone is described by the tangent angle, which is defined as the angle between the axis of symmetry of the tooth and the tangent to the discharge curve, as shown here (Fig.6). It is clear that the location of the most stressed point is the active side.

By applying these principles of discretization to the geometry of the gears, it is possible to generate a fairly structured mesh by decomposing the surface of a tooth into 6 parts and then meshing each of them by quadratic elements with 8 nodes (Figure 5.a).It then becomes easy to refine the mesh locally, without however attributing its global structuring.

The triangular elements are ideal for the discretization of complex geometries, but they can induce an error according to their arrangement compared to the symmetry of the problem. A more structured mesh using quadrilateral elements will therefore be more reliable.

This discretization strategy has therefore been applied and the adjustment of the mesh parameters has been defined so as to obtain convergent results in the region of interest (i.e tooth root).

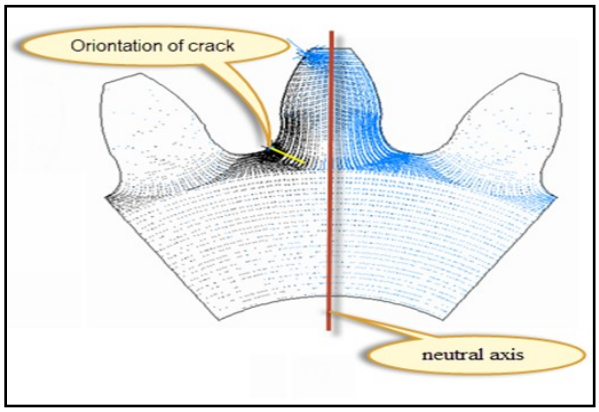


Figure.6**:** Tensile stress (black) & compression stress (blue).

The boundary conditions for the analysis of the growth of the crack are shown in (Fig. 5.b). Initial crack was placed perpendicular to the surface at the location previously determined for initiation in the tension zone of the root of the tooth (Fig. 6).

As an example for 160 N.m (Maximum of load charge), figure 7 shows a comparison between a maximum principal stress and tangentially stress in active side. In figure (7.a), the maximum principal stress reaches a value of 280 Mpa. However,in figure (7.b), the maximum tangential stress reaches a value of 500 Mpa.

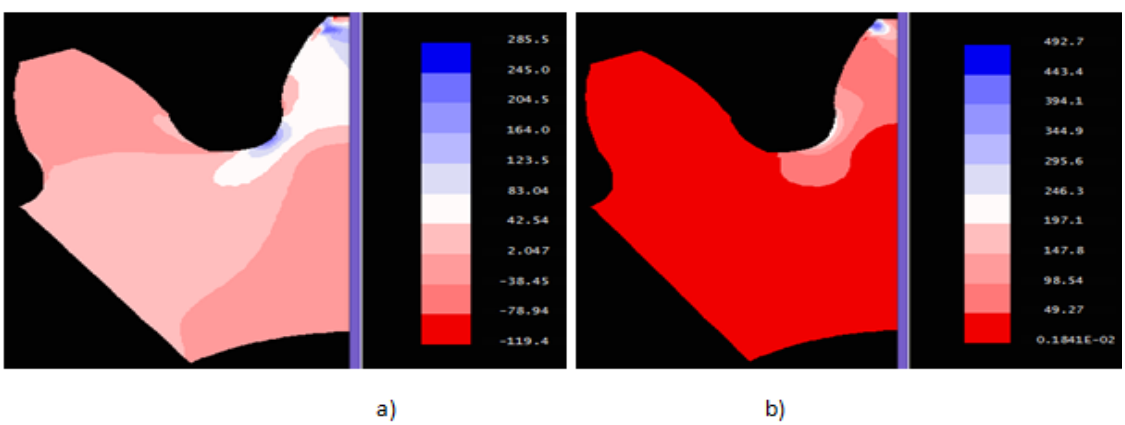


Figure 7: Max stress distribution (MPa): a) principal stress, b) tangentially stress.

Crack initiation threshold, a_{th}, for each material was determined from (Eq. 5),where ΔK_{th} is stress intensity gradient.

$$a_{th} = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{2\sigma_s}\right)^2 \qquad \text{(Eq. 5)}$$

The crack-tip at the root of the tooth is shown in Fig.8, and the distribution of the stress around of the crack-tip is shown in Fig.9. The trajectory and direction of propagation of the crack started from active side to the other side of the toothas shown in Fig.10. The shape of the stress distribution around the crack tip is exactly the same for cracks of all lengths Fig.11.

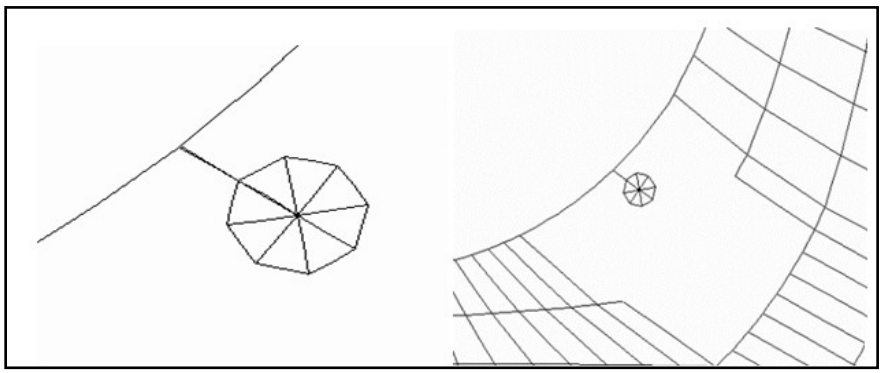


Figure.8**:** Shape and position of the crack at the tooth root

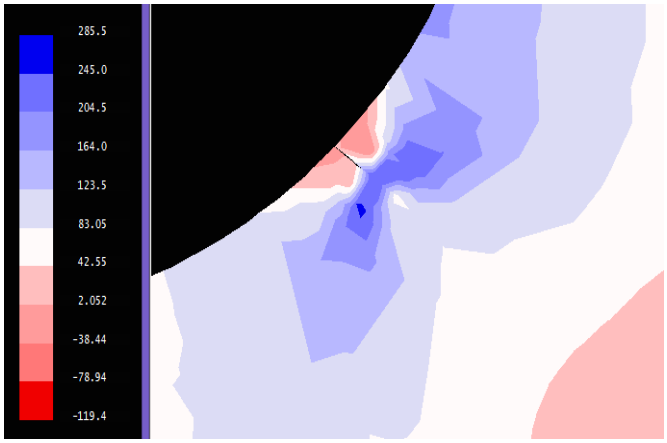


Figure.9 Stress distribution in the Crack tip(MPa).

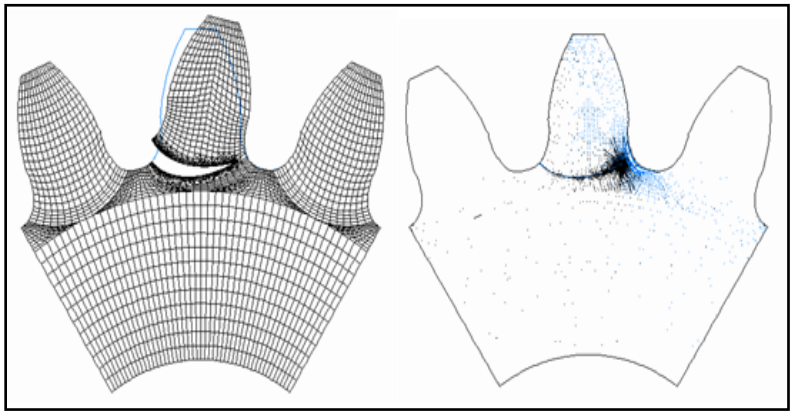
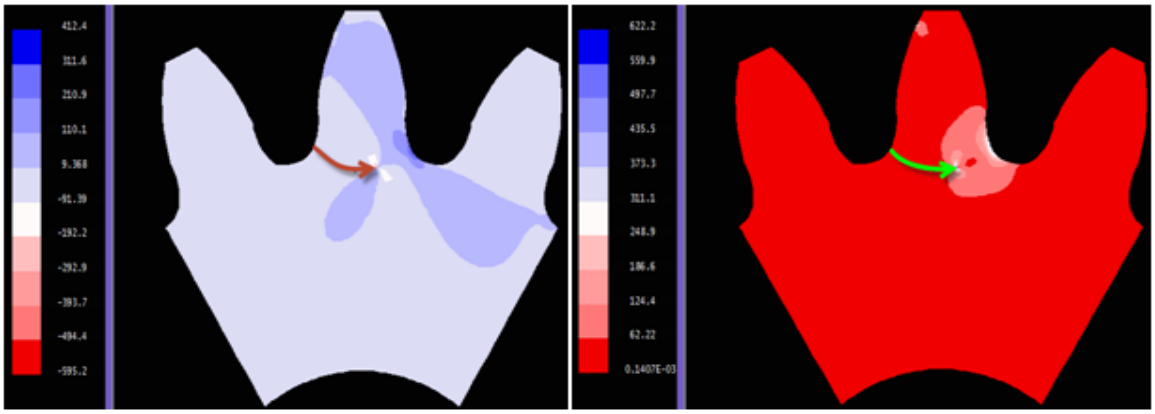


Figure.10 The trajectory of crack propagation.



a) *b)*

Figure.11 a) Distribution of stress (MPa) b) Maximum tangential stress at the crack tip (MPa).

3 RESULTS AND DISCUSSIONS

Compressive and tensile forces develop in the direction of the tooth axis under bending loadsFig.6, induce stresses on the tooth. The maximum compressive stress is found at the non-active side of the tooth while the maximum tensile stress is located at the active side of the tooth. Since the stresses between these two opposing maxima vary linearly, there therefore exists a point on the linear path between them where there is no bending stress. The locus of these points is the neutral axis.

Crack expansion in this case under external forces can generally be divided into two forms:

- Mode I: the crack is opened and extended under tensile stress which is perpendicular to the crack surface.
- Mode II: the crack is slid and extended under shear stress which is parallel to the crack surface and perpendicular to the crack tip (Fig.11.a & b).

For the stress intensity factor (SIF) in our work, we proceed to a study according to several couples, varies between 24 N.m up to 160 N.m, to evaluate the two modes of stress intensity factor 1 and 2. These load types are categorized as Mode I and II, as shown in the Fig.12 for 160 N.m (Maximum of load charge). According to this figure we can clearly notice that the behavior of the SIF versus the crack length in mode 1 have is of exponential form. However, its variation in mode 2 is almost negligible compared to mode 1. Therefore, in our case,it can be concluded that mode 1 is the most dangerous.

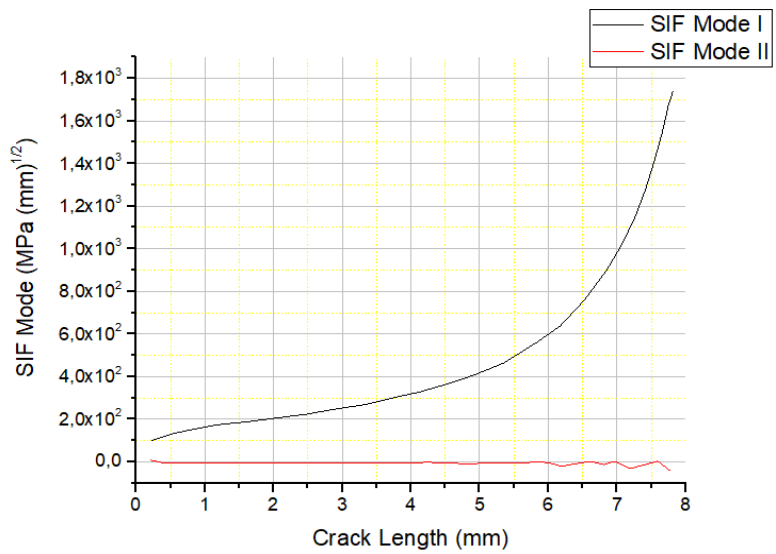


Figure.12: Stress intensity factors versus crack length.

In Fig.13, we can clearly see the increase in the length of the crack as a function of life.It can be notice a gradual variation in crack propagation speed up to 3 mm of the crack length. Beyond this, the propagation speed is stable. Consequently, the variation and evolution of the service life for the propagation (N_p) of the crack of each couple cited before is shown in Fig.14.It can beal so notice that, the crack propagation speed increases up at a value of 3 mm of crack length, if that corresponds to the neutral plane, i.eno presence of bending stress. And from this plane the propagation velocity of the crack is stabilized, because from this point the material loses its resistance.

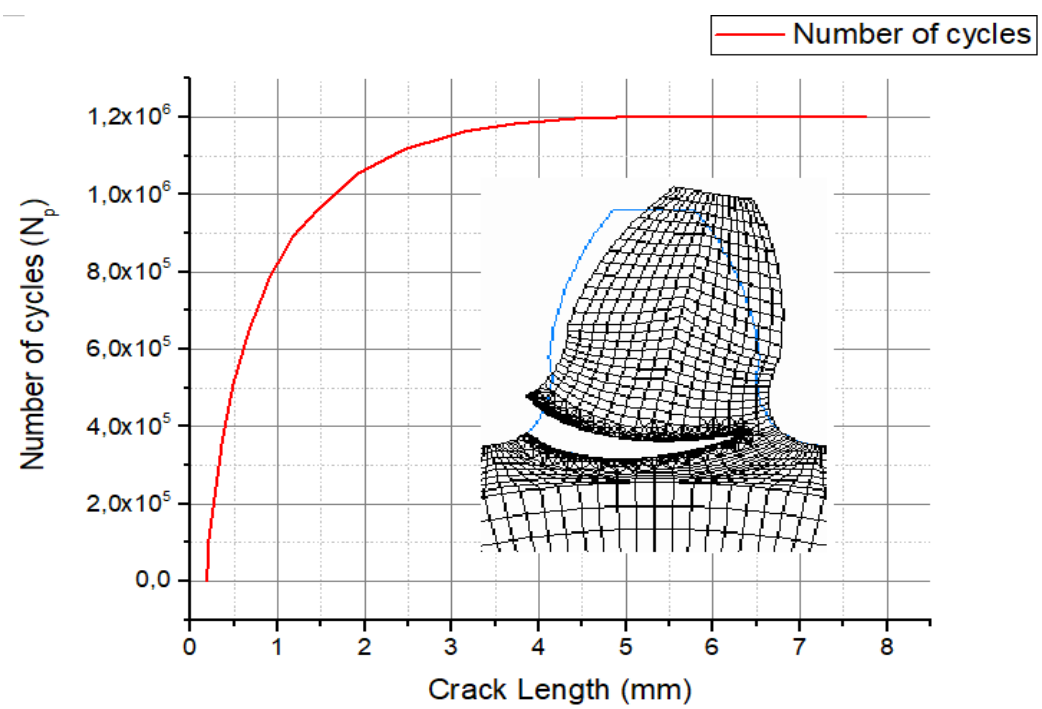


Figure.13 Predicted crack propagation life of gear tooth for “160 N.m load charge”

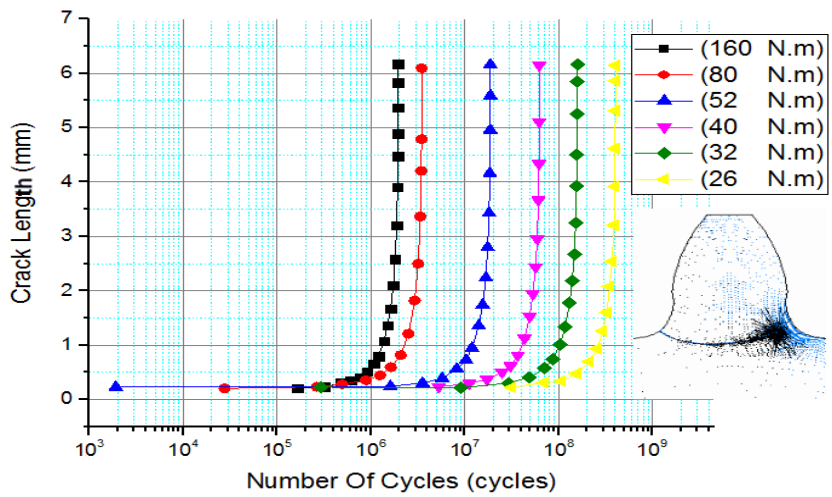


Figure.14 Predicted crack propagation life of gear tooth for different couples applied.

Finally the total life is well calculated, and show with a semi-logarithmic scale in Fig.15 in red color, as a sum of the lifetime of crack initiation and the lifetime of the crack propagation. The black curve is the crack initiation life, as it's shown in the first step (1.1).

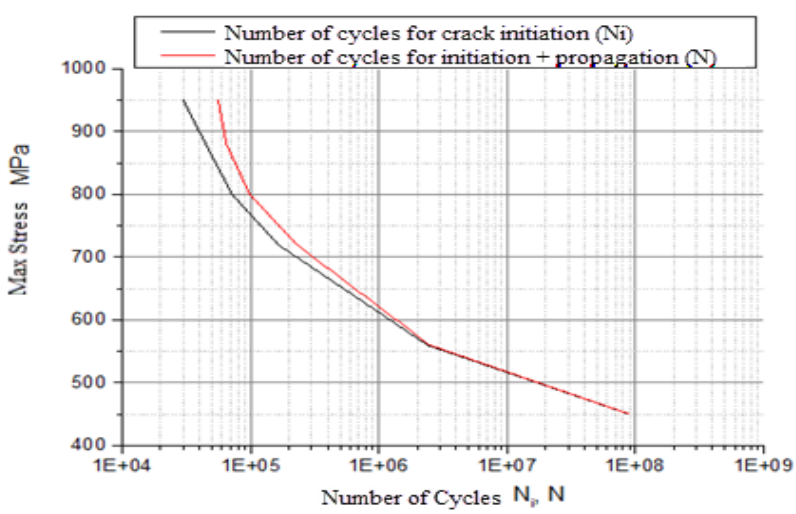


Figure.15 Calculated number of cycle of the bending fatigue.

4. CONCLUSIONS

The paper presents a calculation model to determine the life of bending fatigue of the spur gears and more precisely in its root.The number of load cycles required to fatigue initiation (N_i) and the number of loading cycles for crack propagation (N_p) in the most critical region for bending fatigue was predicted. The proposed model allows the user to determine the entire life of service, for given the appropriate parameters

of the material fatigue. The initiation period of the crack is based on a strain-life analysis using MEF, where it is assumed that the crack is initiated in the area of the maximum principal stress i.e the root of the tooth.The crack propagation of the gear tooth was simulated using the principles of linear elastic fracture mechanics (LEFM).

The functional relationship between the stress intensity factor and the length of the crack $K=f(a)$, which is required for subsequent analysis of thegrowth of fatigue cracks, was also shown.

The model is used to determine the complete lifetime of a steel spur gear combined with high strength 42CrMo4.

The final results of the analysis are presented by MEF. The initiation curve of the crack and the crack propagation, represent the total lifetime.

The results show that at low-stress levels near the fatigue limit, almost the entire life of service is attacked by cracking. This is a very important factor in determining the life of the actual gear because most of gears operate with load conditions in the vicinity of the fatigue limit. The results obtained will be interesting to develop studies among which the development of inspection plans for the studied gears.

NOMENCLATURE

a = crack size

b_i = axial fatigue strength exponent
C = material parameter of Paris equation
E = elastic modulus
G = shear modulus
K' = cyclic strength coefficient
K_c = stress intensity factor critical value
K_{cl} = stress intensity factor when closure occurs
K_I = mode I stress intensity factor
K_{II} = mode II stress intensity factor
K_{th} = stress intensity factor at threshold
ΔK = stress intensity range
ΔK_{eff} = effective stress intensity range
ΔK_{eq} = equivalent stress intensity range
m = material parameter of Paris equation
n' = cyclic strain hardening exponent
N_i = stress cycles for the fatigue crack initiation
N_p = stress cycles for the fatigue crack propagation
R = load ratio
ε_f' = axial fatigue ductility coefficient
v = Poisson ratio
σ_f' = axial fatigue strength coefficient
σ_{FL} = fatigue limit of the tested gear
σ_{FLr} = real fatigue limit
σ_{max} = maximum normal stress on the same plane as ε_a
σ_u = ultimate tensile strength
σ_{ys} = yield stress.
HPSTC = highest point of the single tooth contact
θ_0 = crack-propagation angle

LEMF = linear elastic mechanics of fracture

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